

S-CP.A.4 Construct and interpret two-way frequency tables of data . . . Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

MP 1, MP 3

Objective To construct and use probability models



Support your answer based on the data!



MATHEMATICAL PRACTICES



Lesson Vocabulary

- two-way frequency table
- conditional probability

Think

What is the connection between relative frequency and probability? You can use relative frequency to approximate a probability.



Getting Ready!

The table at the right shows the number of students who passed their driving test as well as whether they took a driver's education class to prepare. What effect, if any, does taking the driver's education class have?

	Passed	Failed	Totals
Took the class	32	7	39
Do not take the class	18	23	41
Totals	50	30	80

In the Solve It, the data is displayed in a *two-way frequency table*. A **two-way frequency table**, or *contingency table*, displays the frequencies of data in two different categories.

Essential Understanding You can use two-way frequency tables to organize data and identify sample spaces to approximate probabilities.



Problem 1 Using a Two-way Frequency Table

Activities The table shows data about student involvement in extracurricular activities at a local high school. What is the probability that a randomly chosen student is a female who is not involved in extracurricular activities?

Extracurricular Activities

	Involved in Activities	Not Involved in Activities	Totals
Male	112	145	257
Female	139	120	259
Totals	251	265	516

To find the probability, calculate the relative frequency.

$$\text{relative frequency} = \frac{\text{females not involved}}{\text{total number of students}} = \frac{120}{516} \approx 0.233$$

The probability that a student is a female who is involved in extracurricular activities is about 23.3%.

**Got It?**

1. The two-way frequency table at the right shows the number of male and female students by grade level on the prom committee. What is the probability that a member of the prom committee is a male who is a junior?

	Male	Female	Totals
Juniors	3	4	7
Seniors	3	2	5
Totals	6	6	12

The probability that an event will occur, given that another event has already occurred is called a **conditional probability**. You can write the conditional probability of event B , given that event A has already occurred as $P(B|A)$. You read $P(B|A)$ as “the probability of event B , given event A .”

**Problem 2 Finding Probability**

Opinion Polls Respondents of a poll were asked whether they were for, against, or had no opinion about a bill before the state legislature that would increase the minimum wage. What is the probability that a randomly selected person is over 60 years old, given that the person had no opinion on the state bill?

Age Group	For	Against	No Opinion	Totals
18–29	310	50	20	380
30–45	200	30	10	240
45–60	120	20	30	170
Over 60	150	20	40	210
Totals	780	120	100	1000

Plan

What part of the table do you need to use?

Since the group you are interested in is the one with no opinion, you only need to look at that column.

The condition that the person selected has no opinion on the minimum-wage bill limits the total outcomes to the 100 people who had no opinion. Of those 100 people, 40 respondents were over 60 years old.

$$P(\text{over 60} | \text{no opinion}) = \frac{40}{100} = 0.4$$

**Got It?**

2. a. What is the probability that a randomly selected person is 30–45 years old, given that the person is in favor of the minimum-wage bill?
- b. **Reasoning** What is the probability that a randomly selected person is not 18–29, given that the person is in favor of the minimum wage bill?



Problem 3 Using Relative Frequencies

Business A company has 150 sales representatives. Two months after a sales seminar, the company vice-president made the table of relative frequencies based on sales results. What is the probability that someone who attended the seminar had an increase in sales?

	Attended Seminar	Did not Attend Seminar	Totals
Increased Sales	0.48	0.02	0.5
No Increase in Sales	0.32	0.18	0.5
Totals	0.8	0.2	1

Think

Given the relative frequencies, how do you find the frequencies?

Multiply the number of sales representatives times the relative frequency for a category.

Method 1 Find frequencies first.

Find the number of people who attended the seminar and had increased sales: $0.48 \cdot 150 = 72$

Find the number of people who attended the seminar: $0.8 \cdot 150 = 120$

Find $P(\text{increased sales} \mid \text{sales seminar})$: $\frac{72}{120} = 0.6$, or 60%

Method 2 Use relative frequencies.

$P(\text{increased sales} \mid \text{sales seminar})$:

$$= \frac{\text{relative frequency of attend seminar and increased sales}}{\text{relative frequency of attended seminar}} = \frac{0.48}{0.8} = 0.6$$

The probability that some who attended the seminar had an increase in sales is 0.6, or 60%.



Got It? 3. What is the probability that a randomly selected sales representative, who did not attend the seminar, did not see an increase in sales?



Lesson Check

Do you know HOW?

Use the two-way frequency table to find the probabilities.

	Supports the Issue	Does not support the issue	Totals
Democrat	24	36	60
Republican	27	33	60
Totals	51	69	120

1. $P(\text{democrat and supports the issue})$
2. $P(\text{democrat} \mid \text{supports the issue})$

Do you UNDERSTAND?



3. **Error Analysis** Using the table at the left, a student calculated the relative frequency of those who do not support the issue, given that they are Republican, as $\frac{33}{33 + 36} \approx 0.478$. What error did the student make?
4. **Vocabulary** What is a two-way frequency table?
5. Suppose A is a female student and B is a student who plays sports. What does $P(B \mid A)$ mean?

A Practice

For Exercises 6–11, use the two-way frequency table below to find the probability of each event.

◀ See Problems 1 and 2.

6. $P(7\text{th-grade girl})$
7. $P(8\text{th-grade boy})$
8. $P(6\text{th-grade girl})$
9. $P(\text{girl} \mid 7\text{th-grade})$
10. $P(6\text{th-grade} \mid \text{boy})$
11. $P(8\text{th-grade} \mid \text{girl})$

Attendance at Soccer Camp

	6th Graders	7th Graders	8th Graders	Totals
Boys	7	6	10	23
Girls	8	7	12	27
Totals	15	13	22	50

Veterinary Medicine For Exercises 12–13, use the table at the right. It shows relative frequencies of treatments that a veterinarian gave dogs and cats during one week.

◀ See Problem 3.

12. When you find the probability that a treatment consists of shots only, given that the vet is treating a cat, how do you limit the sample space?
13. What is the probability that an animal chosen at random is a dog, given that the treatment was shots only?

	Shots only	Shots and Checkup	Totals
Dogs	0.31	0.23	0.54
Cats	0.26	0.20	0.46
Totals	0.57	0.43	1

B Apply

Academic Competition The table at the right shows numbers of participants in an academic competition. Use this information for Exercises 14–18.

14. What is $P(\text{female})$?
15. What is $P(\text{freshman})$?
16. What is $P(\text{female freshman})$?
17. What is $P(\text{female} \mid \text{freshman})$?
18. What is $P(\text{freshman} \mid \text{female})$?

	Male	Female	Totals
Freshmen	3	5	8
Sophomores	6	4	10
Juniors	7	5	12
Seniors	4	6	10
Totals	20	20	40

- © 19. **Think About a Plan** The two-way frequency table at the right shows the number of employees at a factory who have attended a safety workshop and the number who have been injured on the job. What is the probability that a worker from the factory has attended the safety workshop, given that the employee has not been injured on the job?
- Which column is needed for solving this problem?
 - Do you need the total number of workers or the total not injured?

	Injured	Not injured	Totals
Safety workshop	3	36	39
No safety workshop	12	24	36
Totals	15	60	75

- © **20. Reasoning** Recall that two events are independent when the occurrence of one has no effect the other. The table at the right is a frequency table.
- Calculate $P(B)$.
 - Calculate $P(B|A)$.
 - Does the occurrence of A have any effect on the probability of B ? What can you say about events A and B ?
 - Are events C and D independent? Explain.
 - Are events C and F independent? Explain.

	B	D	F	Total
A	7	5	4	16
C	3	4	5	12
E	11	7	2	28
Total	21	16	11	48

- © **21. Writing** What is the sum of the probabilities of all possible outcomes in a probability experiment? How does this relate to the relative frequencies in a contingency table? Explain and give an example.

- 22. Healthcare** The table at the right is a relative frequency distribution for healthy people under the age of 65.

- Copy and complete the table.
- What is the probability of getting the flu, given you are vaccinated?
- What is the probability of getting the flu, given you have not been vaccinated?

Flu Vaccines

	Got the Flu	Did not Get the Flu	Totals
Vaccinated	■	54%	60%
Not Vaccinated	■	■	■
Totals	15%	■	100%



- 23.** When you construct a two-way frequency table, you add across rows and down columns to find values for the totals row and column. The values in the totals column and totals row of the table are called *marginal frequencies*, and the values in the interior of the table are called *joint frequencies*. In the table below, the values 9, 3, 5, and 8 represent the joint frequencies, and the values 14, 11, 12, 13, and 25 represent the marginal frequencies.

	Exam Score $\geq 85\%$	Exam Score $< 85\%$	Totals
Studied more than 4 hours	9	3	12
Studied less than 4 hours	5	8	13
Totals	14	11	25

- Why do you think the values in the interior of the table are called joint frequencies?
- What do the marginal frequencies of the table represent?
- If you replace the joint and marginal frequencies in a two-way frequency table with the respective relative frequencies, what do the values in the totals row and column represent? Use the given table to provide an example.

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

24. The two-way frequency table shows the number of males and females that either support or are against the building of a new mall. What is the probability that a randomly selected person is a female, given that the person supports the new mall? Round to the nearest hundredth.
25. The area of a kite is 150 in.^2 . The length of one diagonal is 50 in. What is the length, in inches, of the other diagonal?
26. What is the x -coordinate of the midpoint of \overline{AB} for $A(-3, 9)$ and $B(-5, -3)$?
27. The segment with endpoints $A(1, 5)$ and $B(2, 1)$ is reflected over $x = 3$ and translated up 2 units and to the right 3 units. What is the x -coordinate of the midpoint of $\overline{A'B'}$?

	For	Against	Totals
Male	62	48	110
Female	78	32	110
Totals	140	80	220

Mixed Review

Find the sum of the interior angle measures of each polygon.

◀ See Lesson 6-1.

28. quadrilateral

29. dodecagon

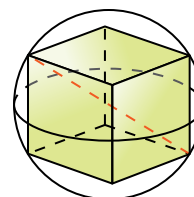
30. 20-gon

31. A cylindrical carton with a radius of 2 in. is 7 in. tall. Assuming no surfaces overlap, what is the surface area of the carton? Round your answer to the nearest square inch.

◀ See Lesson 11-2.

32. A cube with edges 8 cm long fits within a sphere, as shown at the right. The diagonal of the cube is the diameter of the sphere.

- a. Find the radius of the sphere. Leave your answer in simplest radical form.
- b. What is the volume of the sphere?



◀ See Lesson 11-6.

Get Ready! To prepare for Lesson 13-6, do Exercises 33–36.

Find the probability of each event. Use the two-way frequency table.

33. $P(\text{large} \mid \text{red})$

34. $P(\text{red} \mid \text{large})$

35. $P(\text{small} \mid \text{blue})$

36. $P(\text{large} \mid \text{blue})$

	Large	Small	Totals
Blue	17	3	20
Red	8	12	20
Totals	25	15	40